Today’s Objectives:
Students will be able to:
1. Describe the velocity of a rigid body in terms of translation and rotation components.
2. Perform a relative-motion velocity analysis of a point on the body.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Translation and Rotation Components of Velocity
• Relative Velocity Analysis
• Concept Quiz
• Group Problem Solving
• Attention Quiz
As the slider block A moves horizontally to the left with $v_A$, it causes the link CB to rotate counterclockwise. Thus $v_B$ is directed tangent to its circular path.

Which link is undergoing general plane motion? Link AB or link BC?

How can the angular velocity, $\omega$, of link AB be found?
Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.

How can we relate the angular velocities of the various gears in the system?
When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.

\[ \mathbf{dr}_B = \mathbf{dr}_A + \mathbf{dr}_{B/A} \]

Point A is called the base point in this analysis. It generally has a known motion. The x’- y’ frame translates with the body, but does not rotate. The displacement of point B can be written:
The velocity at B is given as: \( \frac{dr_B}{dt} = \frac{dr_A}{dt} + \frac{dr_{B/A}}{dt} \) or
\[
\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}
\]

Since the body is taken as rotating about A,
\[
\mathbf{v}_{B/A} = \frac{dr_{B/A}}{dt} = \mathbf{\omega} \times \mathbf{r}_{B/A}
\]
Here \( \mathbf{\omega} \) will only have a \( \mathbf{k} \) component since the axis of rotation is perpendicular to the plane of translation.
When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.

For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path.

The directions of \( \mathbf{v}_A \) and \( \mathbf{v}_B \) are known since they are always tangent to their paths of motion.
When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground.

Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $\mathbf{v}_B$ has a known direction, e.g., parallel to the surface.
PROCEDURE FOR ANALYSIS

The relative velocity equation can be applied using scalar x and y component equations or via a Cartesian vector analysis.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector \( \mathbf{v}_{B/A} \).

2. Write the equation \( \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \). In the kinematic diagram, represent the vectors graphically by showing their magnitudes and directions underneath each term.

3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.
Vector Analysis:

1. Establish the fixed x - y coordinate directions and draw the kinematic diagram of the body, showing the vectors \( \mathbf{v}_A, \mathbf{v}_B, \mathbf{r}_{B/A} \) and \( \omega \). If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in Cartesian vector form (CVN) and substitute them into \( \mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} \). Evaluate the cross product and equate respective \( i \) and \( j \) components to obtain two scalar equations.

3. If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.
Given: Roller A is moving to the right at 3 m/s.

Find: The velocity of B at the instant $\theta = 30^\circ$.

Plan:

1. Establish the fixed x - y directions and draw a kinematic diagram of the bar and rollers.

2. Express each of the velocity vectors for A and B in terms of their $i, j, k$ components and solve $\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$. 

EXAMPLE 1
Solution:

Kinematic diagram:

Express the velocity vectors in CVN
\[ \mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} \]
\[ -\mathbf{v}_B = 3 \mathbf{i} + [\omega \mathbf{k} \times (-1.5 \cos 30 \mathbf{i} + 1.5 \sin 30 \mathbf{j})] \]
\[ -\mathbf{v}_B = 3 \mathbf{i} - 1.299 \omega \mathbf{j} - 0.75 \omega \mathbf{i} \]

Equating the \( \mathbf{i} \) and \( \mathbf{j} \) components gives:
\[ 0 = 3 - 0.75 \omega \]
\[ -\mathbf{v}_B = -1.299 \omega \]

Solving: \( \omega = 4 \text{ rad/s} \) or \( \omega = 4 \text{ rad/s} \mathbf{k} \)
\[ \mathbf{v}_B = 5.2 \text{ m/s} \] or \( \mathbf{v}_B = -5.2 \text{ m/s} \mathbf{j} \)
**EXAMPLE II**

**Given:** Crank rotates OA with an angular velocity of 12 rad/s.

**Find:** The velocity of piston B and the angular velocity of rod AB.

**Plan:**

Notice that point A moves on a circular path. The directions of $v_A$ is tangent to its path of motion. Draw a kinematic diagram of rod AB and use

$$v_B = v_A + \omega_{AB} \times r_{B/A}.$$
EXAMPLE II (continued)

Solution:

Kinematic diagram of AB:

Since crack OA rotates with an angular velocity of 12 rad/s, the velocity at A will be:

\[ \mathbf{v}_A = -0.3(12) \mathbf{i} = -3.6 \mathbf{i} \text{ m/s} \]

Rod AB. Write the relative-velocity equation:

\[ \mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} \]

\[ \mathbf{v}_B \mathbf{j} = -3.6 \mathbf{i} + \omega_{AB} \mathbf{k} \times (0.6\cos30 \mathbf{i} - 0.6\sin30 \mathbf{j}) \]

\[ \mathbf{v}_B \mathbf{j} = -3.6 \mathbf{i} + 0.5196 \omega_{AB} \mathbf{j} + 0.3 \omega_{AB} \mathbf{i} \]

By comparing the \( \mathbf{i}, \mathbf{j} \) components:

\[ \mathbf{i}: \ 0 = -3.6 + 0.3 \omega_{AB} \quad \Rightarrow \quad \omega_{AB} = 12 \text{ rad/s} \]

\[ \mathbf{j}: \ \mathbf{v}_B = 0.5196 \omega_{AB} \quad \Rightarrow \quad \mathbf{v}_B = 6.24 \text{ m/s} \]
GROUP PROBLEM SOLVING

**Given:** The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. The link AB is rotating at $\omega_{AB} = 4 \text{ rad/s}$.

**Find:** The velocity of the slider block C when $\theta = 60^\circ$.

**Plan:** Notice that link AB rotates about a fixed point A. The directions of $\mathbf{v}_B$ is tangent to its path of motion. Draw a kinematic diagram of rod BC. Then, apply the relative velocity equations to the rod and solve for unknowns.
GROUP PROBLEM SOLVING
(continued)

Solution:

Since link AB is rotating at $\omega_{AB} = 4 \text{ rad/s}$, the velocity at point B will be:

$v_B = 4 \times 300 = 1200 \text{ mm/s}$

At $\theta = 60^\circ$, $v_B = -1200 \cos 30 \hat{i} + 1200 \sin 30 \hat{j}
= (-1039 \hat{i} + 600 \hat{j}) \text{ mm/s}$

Draw a kinematic diagram of rod BC.

Notice that the slider block C has a horizontal motion.
GROUP PROBLEM SOLVING (continued)

Solution continued:

Apply the relative velocity equation in order to find the velocity at C.

\[ \mathbf{v}_C = \mathbf{v}_B + \mathbf{\omega}_{BC} \times \mathbf{r}_{C/B} \]

\[ \mathbf{v}_C \mathbf{i} = ( -1039 \mathbf{i} + 600 \mathbf{j} ) \]
\[ + \mathbf{\omega}_{BC} \mathbf{k} \times ( -125 \cos 45 \mathbf{i} + 125 \sin 45 \mathbf{j} ) \]

\[ \mathbf{v}_C \mathbf{i} = ( -1039 - 125 \mathbf{\omega}_{BC} \sin 45 )\mathbf{i} + ( 600 - 125 \mathbf{\omega}_{BC} \cos 45 )\mathbf{j} \]

Equating the \( i \) and \( j \) components yields:

\[ \mathbf{v}_C = -1039 - 125 \mathbf{\omega}_{BC} \sin 45 \]
\[ 0 = 600 - 125 \mathbf{\omega}_{BC} \cos 45 \]

\[ \mathbf{\omega}_{BC} = 4.8 \text{ rad/s} \]
\[ \mathbf{v}_C = -1639 \text{ mm/s} = 1639 \text{ mm/s} \]
16-71.

The two-cylinder engine is designed so that the pistons are connected to the crankshaft $BE$ using a master rod $ABC$ and articulated rod $AD$. If the crankshaft is rotating at $\omega = 30$ rad/s, determine the velocities of the pistons $C$ and $D$ at the instant shown.
End of the Lecture

Let Learning Continue